A non-universal transition to asymptotic freedom in low-energy quantum gravity

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Abstract

The model of low-energy quantum gravity by the author has the property of asymptotic freedom at very short distances. The character of transition to asymptotic freedom is studied here. It is shown that this transition is not universal, but the one obeys the scaling rule: the range of this transition in units of $r/E^{1/2}$, where r is a distance between particles and E is an energy of the screening particle, is the same for any micro-particle. This range for a proton is between $10^{-11} - 10^{-13}$ meter, while for an electron it is approximately between $10^{-13} - 10^{-15}$ meter.

1 Introduction

Recently, it was shown by the author [1] that asymptotic freedom appears at very short distances in the model of low-energy quantum gravity [2]. In this case, the screened portion of gravitons tends to the fixed value of 1/2, that leads to the very small limit acceleration of the order of 10^{-13} m/s^2 of any screened micro-particle. While asymptotic freedom of strong interactions [3, 4] is due to the anti-screening effect of gluons, the gravitational one is caused by the external character of graviton flux and the limited rise of

the screened portion at ultra short distances. In this paper, I consider how a transition occurs from the inverse square law to almost full asymptotic freedom. The most important property of this transition is its non-universality: for different particles it takes place in different distance ranges, and the order of these ranges is terribly far from the Planck scale where one usually waits of manifestations of quantum gravity effects.

2 The screened portion of gravitons at very short distances

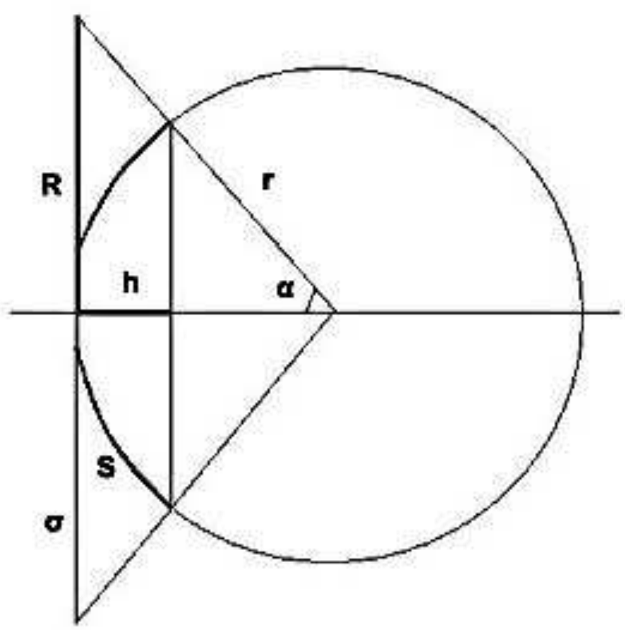

Figure 1: To the computation of the screened portion of gravitons at small distances: σ is the cross-section, S is a square of the spherical segment of a hight h.

In the model of low-energy quantum gravity [2], the condition of big

distances:

$$\sigma(E_2, <\epsilon_2>) \ll 4\pi r^2. \tag{1}$$

should be accepted to have the Newton law of gravitation. I use here the notations of [2]: $\sigma(E_2, <\epsilon_2>)$ is the cross-section of interaction of graviton pairs with an average pair energy $<\epsilon_2>$ with a particle having an energy

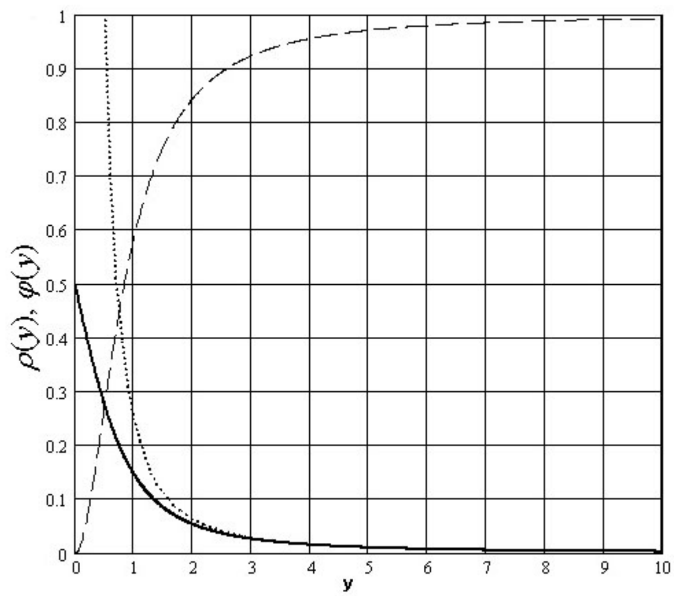

Figure 2: Graphs of the functions $\rho(y)$ (*solid*), $\rho(y)_{cl}$ (*dot*), $\varphi(y)$ (*dash*) .

E_2, r is a distance between particles 1 and 2. As it was shown in [5], the equivalence principle should be broken at distances $\sim 10^{-11}\ m$, when the condition (1) is violated for a proton-mass particle. The ratio

$$\sigma(E_2, <\epsilon_2>)/4\pi r^2. \tag{2}$$

describes the screened portion of gravitons for a big distance r. For small r, let us consider Fig. 1, where $R = (\sigma(E_2, <\epsilon_2>)/\pi)^{1/2}$, S is the screening area (the square of the spherical segment of the hight h), and α is an angle for which $\cot\alpha = r/R \equiv y$. Then we get for S :

$$S = 2\pi r^2(1 - y/(1+y^2)^{1/2}), \tag{3}$$

and it is necessary to replace the ratio (2) by the following one:

$$\rho(y) \equiv S/4\pi r^2 = (1 - y/(1+y^2)^{1/2})/2. \tag{4}$$

I rewrite (2) as $\rho(y)_{cl} \equiv 1/4y^2$, and I introduce the ratio of these functions:

$$\varphi(y) \equiv \rho(y)/\rho(y)_{cl} = 2y^2(1 - y/(1+y^2)^{1/2}). \tag{5}$$

In Fig. 2, the behavior of the functions $\rho(y)$, $\rho(y)_{cl}$, $\varphi(y)$ is shown. The upper limit of $\rho(y)$ by $y \to 0$ is equal to 1/2; namely this property of the function leads to asymptotic freedom [1].

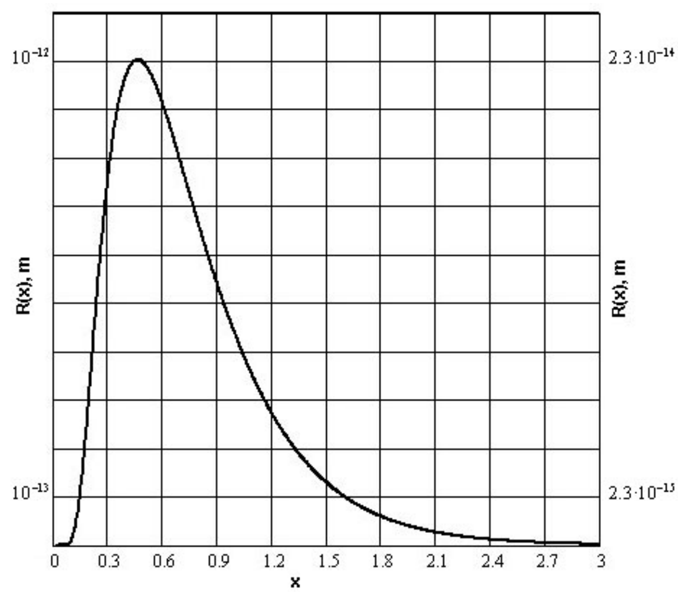

Figure 3: The function $R(x)$ for the two cases: $E_2 = m_p c^2$ (the left logarithmic vertical scale) and $E_2 = m_e c^2$ (the right logarithmic vertical scale).

In this model, the cross-section $\sigma(E_2, <\epsilon_2>)$ is equal to [2]:

$$\sigma(E_2, <\epsilon_2>) = \frac{DkTE_2 2x(1-\exp(-(\exp(2x)-1)^{-1}))(\exp(2x)-1)^{-2}}{\exp((\exp(2x)-1)^{-1})\exp((\exp(x)-1)^{-1})}, \tag{6}$$

where $T = 2.7K$ is the temperature of the graviton background, $x \equiv \hbar\omega/kT$, $\hbar\omega$ is a graviton energy, the new constant D has the value: $D = 0.795 \cdot 10^{-27} m^2/eV^2$. The quantity $R(x)$ has been computed for the two cases (see Fig. 3): $E_2 = m_p c^2$ (the left vertical axis on Fig. 3) and $E_2 = m_e c^2$ (the right vertical axis on Fig. 3), where m_p and m_e are masses of a proton and of an electron, correspondingly.

3 A non-universal transition to asymptotic freedom

To find the net force of gravitation $F = F_2/2$ at a small distance r, we should replace the factor $\sigma(E_2, <\epsilon_2>)/4\pi r^2$ in Eq. (31) of [2] with the more exact factor $S/4\pi r^2$. Then we get:

$$F(r) = \frac{4}{3} \cdot \frac{D(kT)^5 E_1}{\pi^2 \hbar^3 c^3} \cdot g(r), \tag{7}$$

where E_1 is an energy of particle 1, and $g(r)$ is the function of r:

$$g(r) \equiv \int_0^\infty \frac{x^4(1 - \exp(-(\exp(2x) - 1)^{-1}))(\exp(2x) - 1)^{-3}}{\exp((\exp(2x) - 1)^{-1}) \exp((\exp(x) - 1)^{-1})} \cdot \rho(y) dx, \tag{8}$$

where $y = y(r, x) = r/R(x)$. By $r \to 0$, this function's limit for any E_2

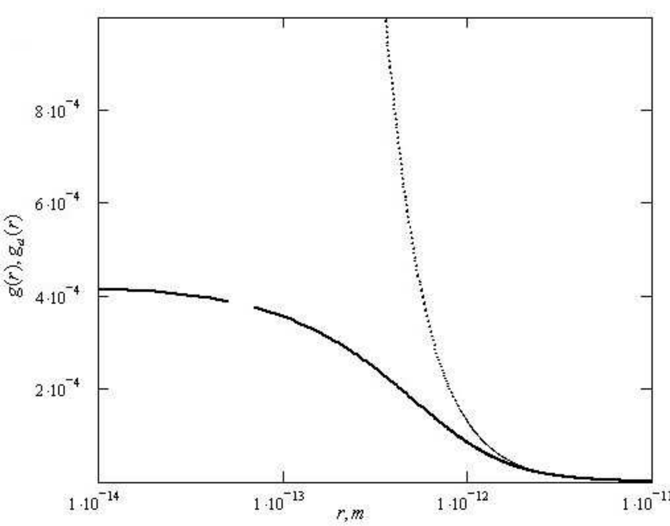

Figure 4: Graphs of the functions $g(r)$ (solid) and $g_{cl}(r)$ (dot) for the case $E_2 = m_p c^2$.

is: $g(r) \to I_5 = 4.24656 \cdot 10^{-4}$ [1]. Because breaking the inverse square law is described with this new function, it will be convenient to introduce the function $g_{cl}(r) \propto 1/r^2$ which differs from $g(r)$ only with the replacement: $\rho(y) \to \rho_{cl}(y)$. Graphs of these two functions, $g(r)$ and $g_{cl}(r)$, are shown

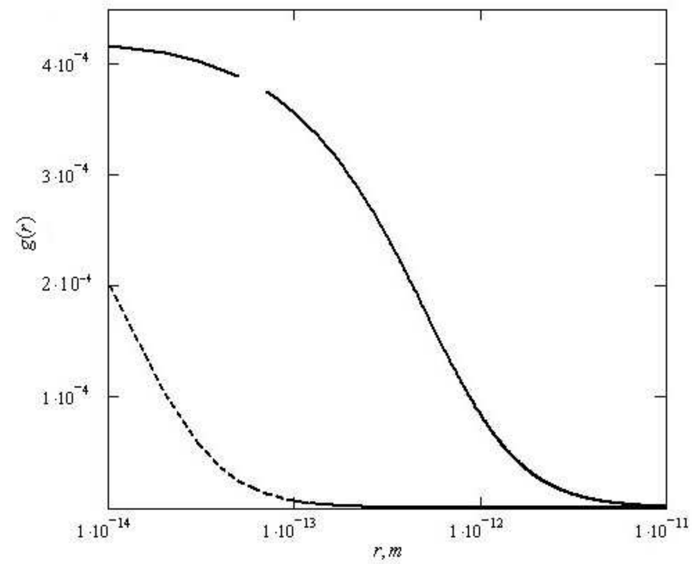

Figure 5: Different transition to the limit value of the function $g(r)$ by $E_2 = m_pc^2$ (solid) and by $E_2 = m_ec^2$ (dot).

in Fig. 4 for the case $E_2 = m_pc^2$. For comparison, graphs of the function $g(r)$ are shown in Fig. 5 for the following different energies: $E_2 = m_pc^2$ and $E_2 = m_ec^2$. The functions have the same limit by $r \to 0$, but the most interesting thing is their different transition to this limit when r decreases.

To underline this non-universal behavior, we can compute the ratio:

$$\eta(r) \equiv g(r)/g_{cl}(r), \tag{9}$$

which aims to unity by big r. Graphs of this function $\eta(r)$ are shown in Fig. 6 for the same energies: $E_2 = m_pc^2$ and $E_2 = m_ec^2$. As we see in this picture, the range of transition for a proton is between $10^{-11} - 10^{-13}$ meter, while for an electron it is between $10^{-13} - 10^{-15}$ meter. So as $y(r, x) = r/R(x) \propto r/E_2^{0.5}$, it is obvious that the functions $g(r/E_2^{0.5})$ and $\eta(r/E_2^{0.5})$ are universal for any energy E_2 of a micro-particle. This scaling law means, for example, that if deviations from the inverse square law begins for a proton at $r_0 \sim 10^{-11}$ m, then for a particle with an energy of E_{2x}, the same deviations appear at $r_{0x} = r_0 \cdot (E_{2x}/m_pc^2)^{0.5}$.

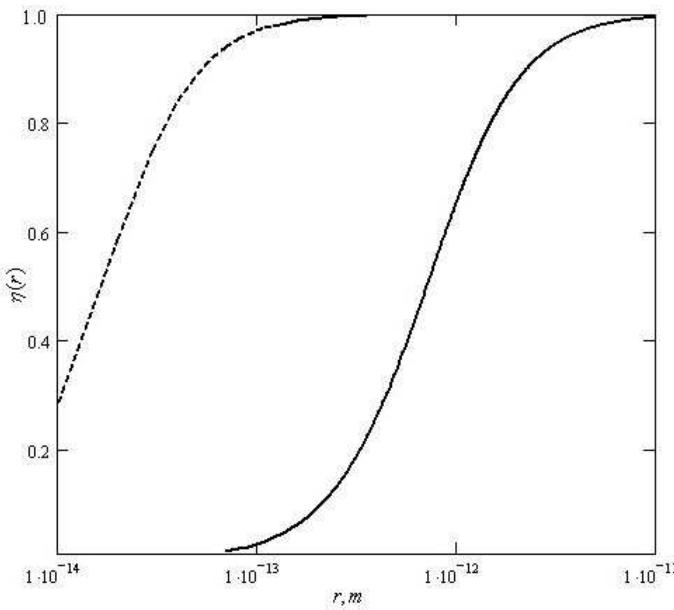

Figure 6: A non-universal transition to the limit value of unity of the function $\eta(r)$ by $E_2 = m_p c^2$ (solid) and by $E_2 = m_e c^2$ (dot).

4 Conclusion

The considered model has the two unexpected properties: asymptotic freedom and a non-universal transition to it. As distinct from QCD, at very small distances the attractive force of gravitation doesn't decrease when $r \to 0$, instead, it remains only finite and very small - but its limit value is the maximal possible one. Perhaps, it would be better to say that gravity between micro-particles gets a saturation at short distances in this model.

References

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